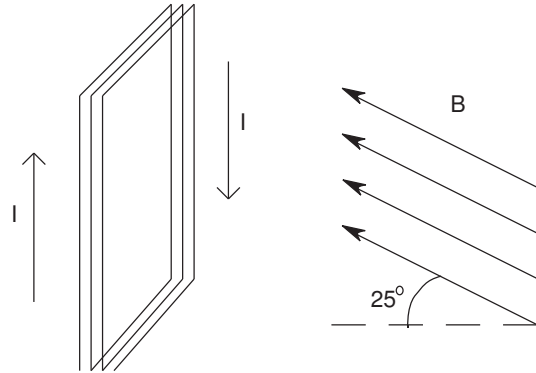# Homework on Current Loops in Magnetic Fields Solutions

1. A rectangular loop of 10 turns with width 35 cm and length 60 cm carrying a current of 3.4 A is in a magnetic field of 2800 Gauss angled as shown in the diagram to the right. Find the magnitude of the torque on the loop and indicate the direction of the torque vector.

The loop will rotate clockwise, so the torque vector will point into the page (by the right hand rule).

The magnitude is:

$$\tau = |\mathbf{\mu} \times \mathbf{B}| = \mu B \sin\theta = NAIB \sin\theta = (10)(0.35 \times 0.6)(3.4)(0.28)\sin(25°) = 0.845 \, N\,m$$

2. A 10 kg weight is hung from a frictionless, massless pulley as shown. Attached to the axis of the pulley is a massless circuit with a resistance of 2 ohms and a battery producing 500 V of EMF as shown. A uniform magnetic field of 5000 Gauss points to the left.
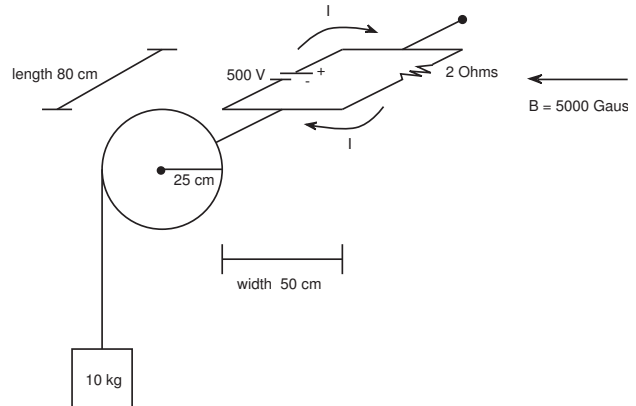
a. Find the magnitude and direction of the magnetic moment of the loop.

By the right hand rule, looking at the current loop, the magnetic moment points down.

The current can be found using Ohm's law:

$$I = \frac{V}{R} = \frac{500}{2} = 250A$$

Then the magnetic moment has a magnitude:

$$\mu = NAI = (1)(.5 \times .8)(250) = 100 \, A\,m^2$$

b. Find the magnitude and direction of the magnetic torque on the loop. Also find the gravitational torque from the hanging mass.

The magnetic torque will rotate the apparatus clockwise and so points into the page, it has a magnitude of

$$\tau = |\boldsymbol{\mu} \times \mathbf{B}| = \mu B \sin\theta = (100)(.5)\sin 90° = 50 N\,m$$

There is a gravitational force of magnitude Mg pulling down on the mass, which exerts a torque on the wheel spinning the apparatus counterclockwise. By the right hand rule, the torque points out of the page.

$$\tau = |\mathbf{r} \times \mathbf{F}_g| = rMg \sin\theta = (0.25)(10)(9.81)\sin 90° = 24.525 \ N\,m$$

The total torque is then 25.475 N m in the clockwise direction, so the magnetic torque will overcome gravity pull the hanging mass up.

c. When the loop rotates to the upright position, how much magnetic potential energy is lost? How much gravitational potential energy is gained?

Magnetic Potential Energy:

$$U_f - U_i = -\boldsymbol{\mu}_i \cdot \mathbf{B} + \boldsymbol{\mu}_f \cdot \mathbf{B} = -\mu B \cos 0° + \mu B \cos 90° = -\mu B = -50J$$

So we have lost 50 J of magnetic potential energy

Gravitational Potential Energy:

Note that the height increase of the mass is one quarter of the circumference of the wheel.

$$U_f - U_i = mgh_f - mgh_i = mg\left(h_f - h_i\right) = mg\Delta h = mg\left(\frac{1}{4}\right)2\pi r = \frac{\pi mgr}{2} = \frac{\pi(10)(9.81)(0.25)}{2} = 38.5J$$

So we have gained 38.5 J of gravitational potential energy.

d. What is the kinetic energy of the block? What velocity of the block? What is the angular velocity of the current loop?

Overall, we lost 11.5 J of potential energy, which have now been transferred into kinetic energy of the hanging mass. Since the apparatus is idealized to have no mass, we do not need to worry about any kinetic energy involved with its spinning.

Then relating kinetic energy to velocity,

$$K = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2K}{M}} = \sqrt{\frac{2(11.5)}{(10)}} = 1.52 \; m/s$$

Then the angular velocity is

$$\omega = \frac{v}{r} = \frac{1.52}{.25} = 6.07 \; rad/s$$

e. Suppose it was possible to switch the direction of the EMF source on the circuit. How could one time the switching so that the apparatus would continue to rotate and lift the mass?

When the loop passes the vertical position, the magnetic moment begins to up, and so the magnetic torque is then clockwise (to return the two vectors to the parallel position). If we reversed the battery, this would reverse the current flow and hence the magnetic moment, and thus the direction of the torque. Therefore, if we reverse the battery every time the loop moves to an upright position, the magnetic torque will always rotate the apparatus in a clockwise fashion.