TeachEngineering

Fluids: Archimedes’ Principle, Pascal’s Law, Bernoulli’s Principle

Fluids and Buoyant Force

- **Fluids**: Matter that flows (liquid and gas).
- **Mass Density**: Mass per unit volume of a substance. It is often represented by the Greek letter ρ (rho).

$$\rho = \frac{m}{V}$$

- **Buoyant Force**: The upward force on objects that are partially or completely submerged in fluids.

Archimedes' Principle

“Any object completely or partially submerged in a fluid experiences an upward force equal in magnitude to the weight of the fluid displaced by the object.”

- **Archimedes’ Principle**

For Floating Objects

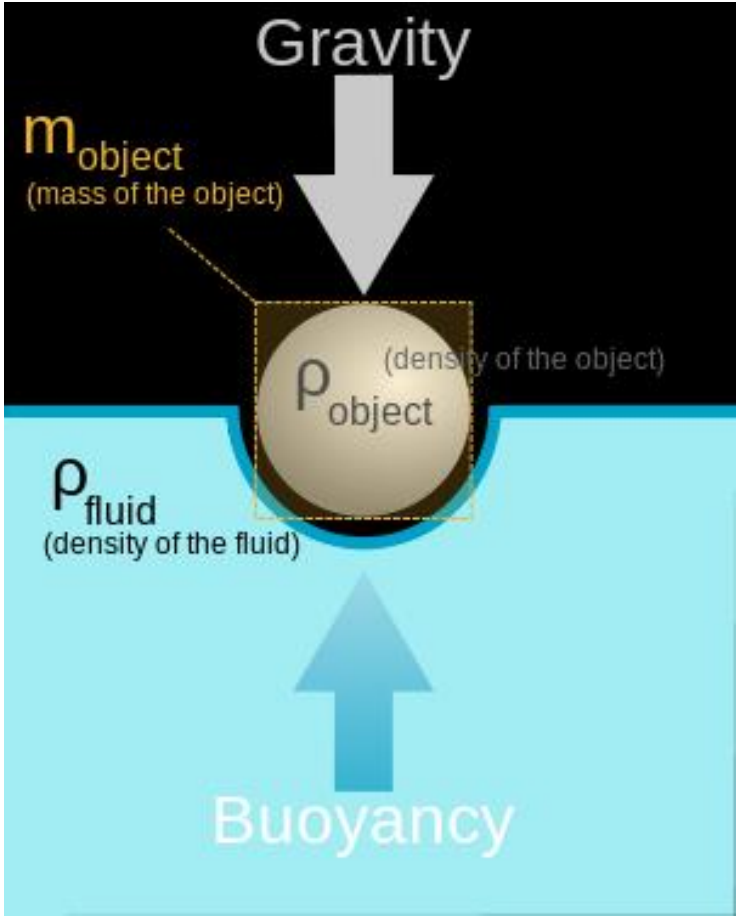
Buoyant Force:

$$F_B = F_{g\,(displaced)} = m_f\,g$$

where m_f = mass of fluid displaced

For Floating Objects:

$$F_B = F_{g\,(object)} = m_o\,g$$



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Archimedes' Principle:

The buoyant force is equal to the weight of the displaced water.

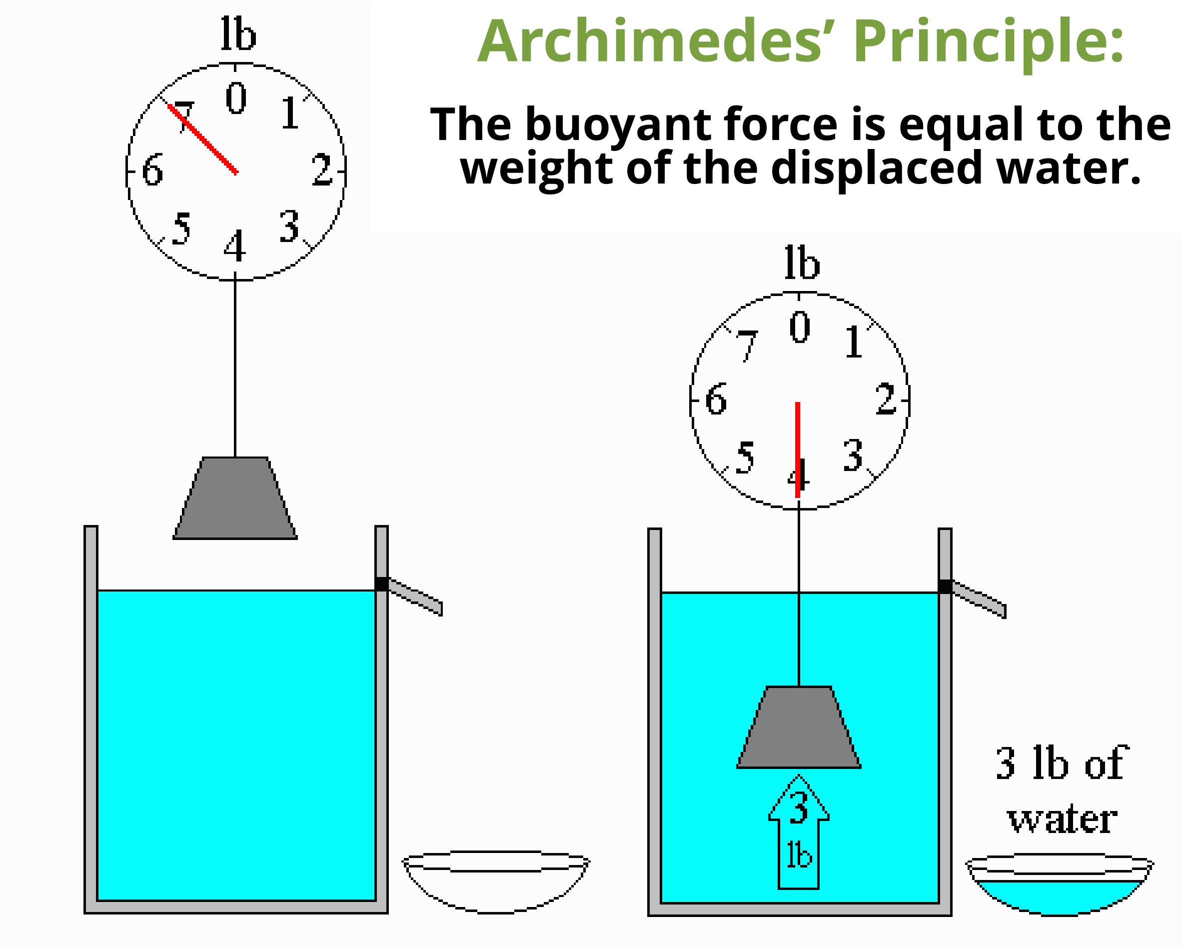
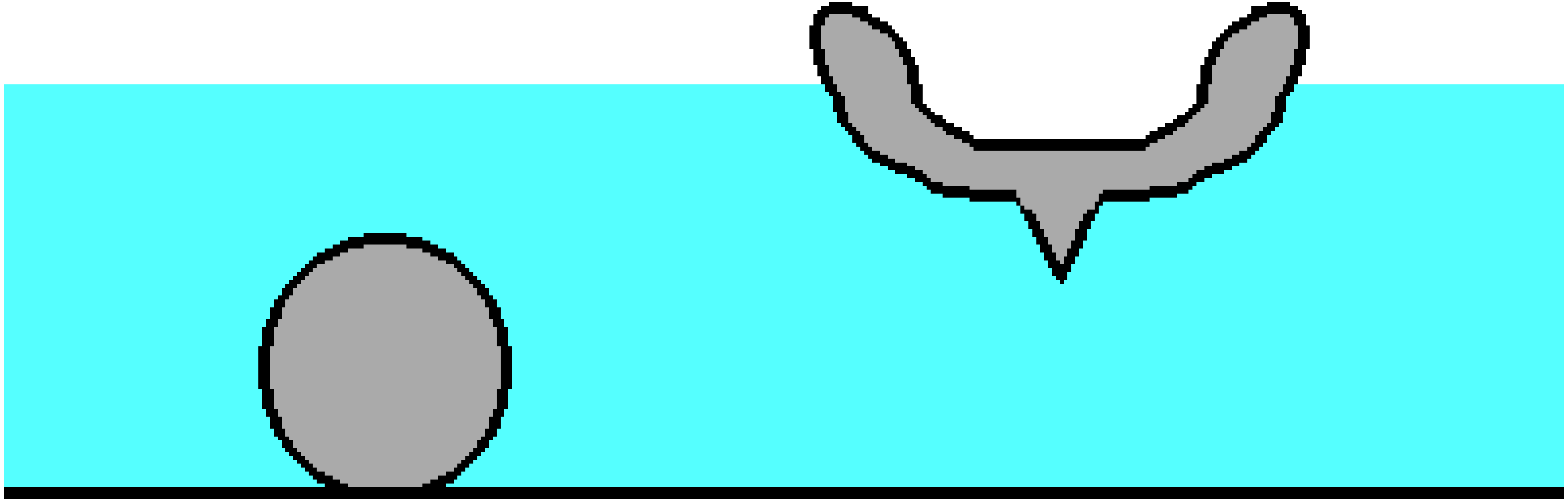


Image source: Bradley W. Carroll. Used with permission.
http://physics.weber.edu/carroll/archimedes/principle.htm

ball: **displaced water weighs less than the ball**

hull: **displaced water weight equals hull weight**



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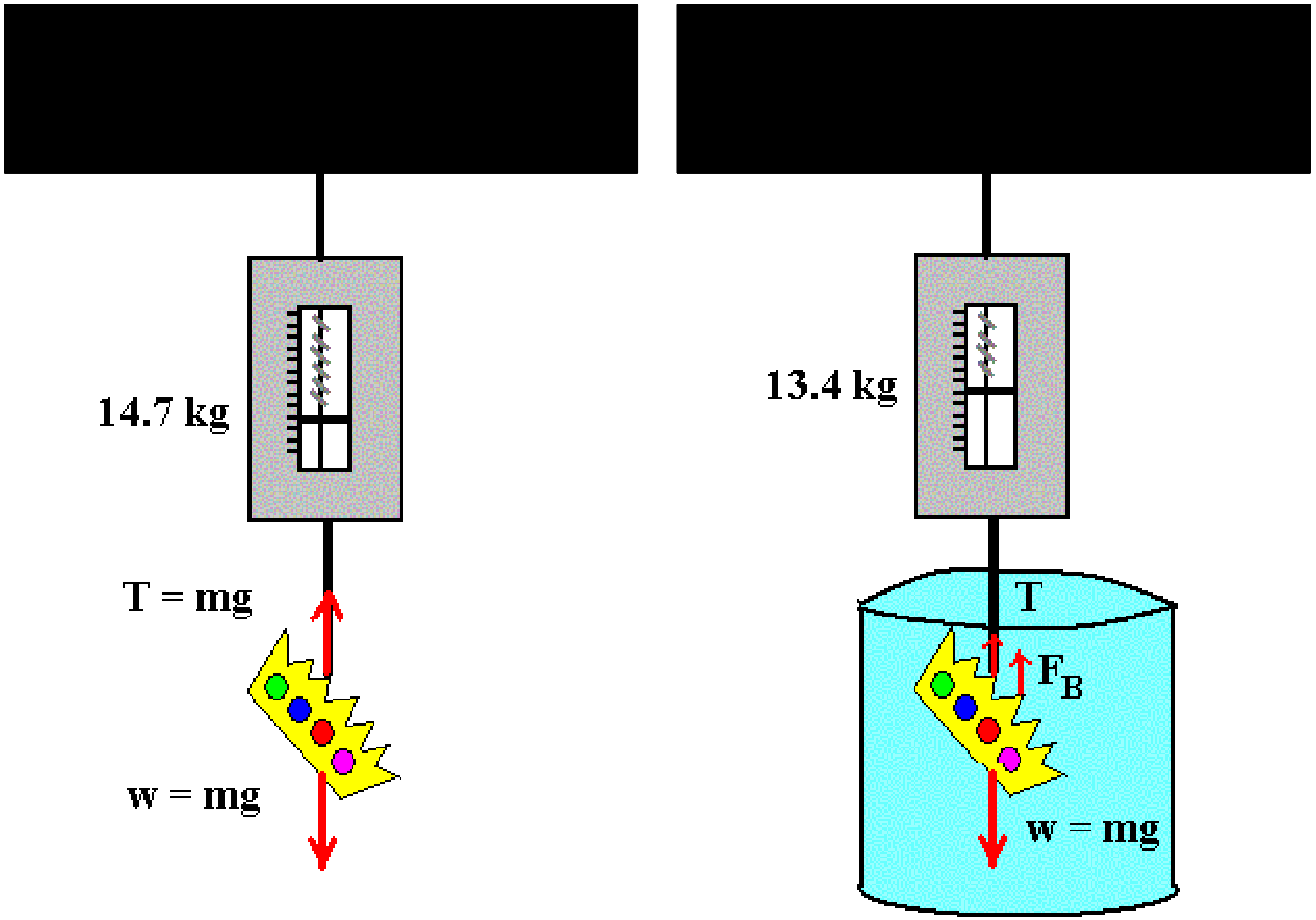
http://physics.weber.edu/carroll/archimedes/principle.htm

Buoyant Force

Buoyant force is also equal to the difference between the weight of an object in air and weight of an object in fluid.

$$F_B = W_{air} - W_{fluid}$$

In other words, the apparent loss in weight of a body immersed in a fluid is equal to the weight of the displaced fluid.



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Other Relationships

Net force (F_{net}) is the object's apparent weight:

$$F_{net} = F_B - F_{g\,(object)}$$

$$F_{net} = (\rho_f v_f - \rho_o v_o)\, g$$

where: $m = \rho v$

In solving buoyancy problems, the following derived expression is used:

$$\frac{F_{g\,(object)}}{F_B} = \frac{\rho_0}{\rho_f}$$

Pascal's Law

Pressure

Pressure is a measure of how much force is applied over a given area.

$$P = \frac{F}{A}$$

units:

1 Pa (Pascal) = 1 N/m^2

1 atm = 105 Pa

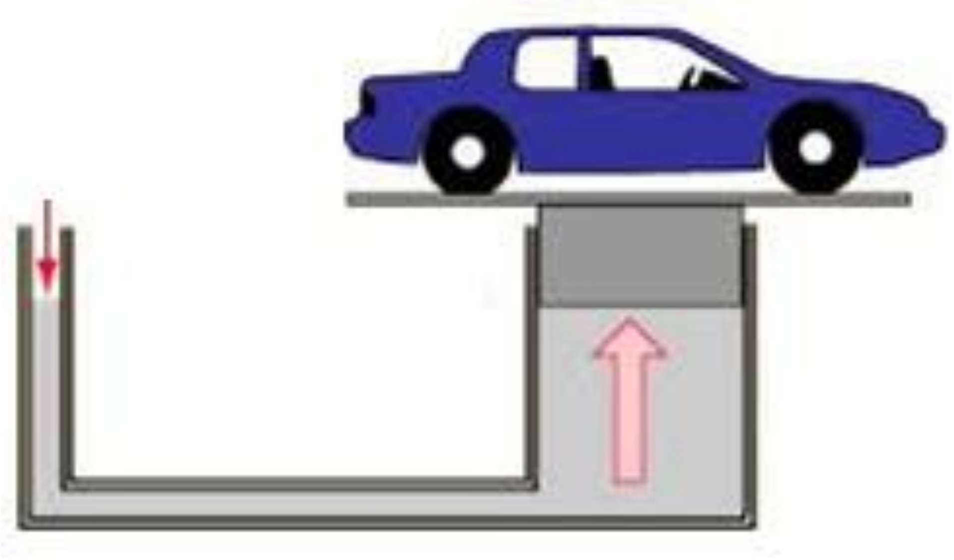
“Pressure applied to a fluid in a closed container is transmitted equally to every point of the fluid and to the walls of the container.”

- Pascal’s Law

- Pressure applied anywhere to a fluid causes a force to be transmitted equally in all directions.
- Change in pressure disperses equally throughout the fluid.
- Force acts at right angles to any surface in contact with the fluid.

$A_1 = 1\ m^2$
$F_1 = 10$ N
$P_1 =$ ___?

$A_2 = 10\ m^2$
$P_2 =$ ____?
$F_2 =$ ____?



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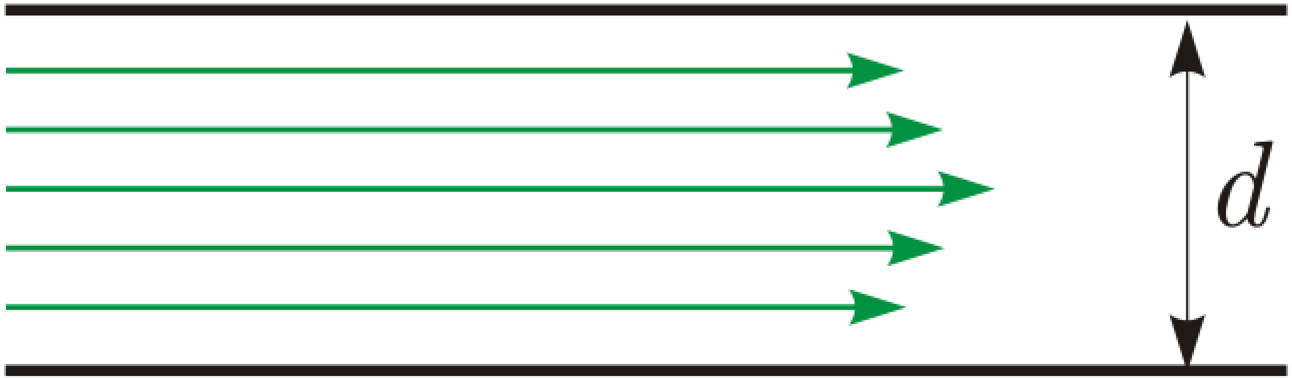
Bernoulli’s Principle

Types of Fluid Flow

- **Laminar:** When fluid particles move along the same smooth path. The path is called a streamline.



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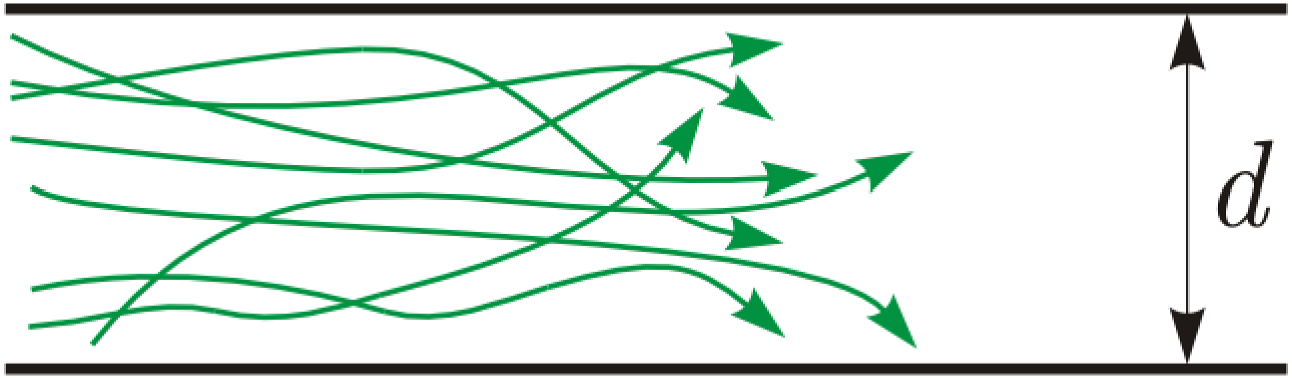
Types of Fluid Flow

- **Turbulent:** When fluid particles flow irregularly causing changes in velocity. They form eddy currents.



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“The pressure in a fluid decreases as the fluid’s velocity increases.”

- Bernoulli’s Principle

Continuity equation:

$$A_1v_1 = A_2v_2$$

Bernoulli’s equation:

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

Bernoulli's equation at different points in a horizontal pipe:

$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$$

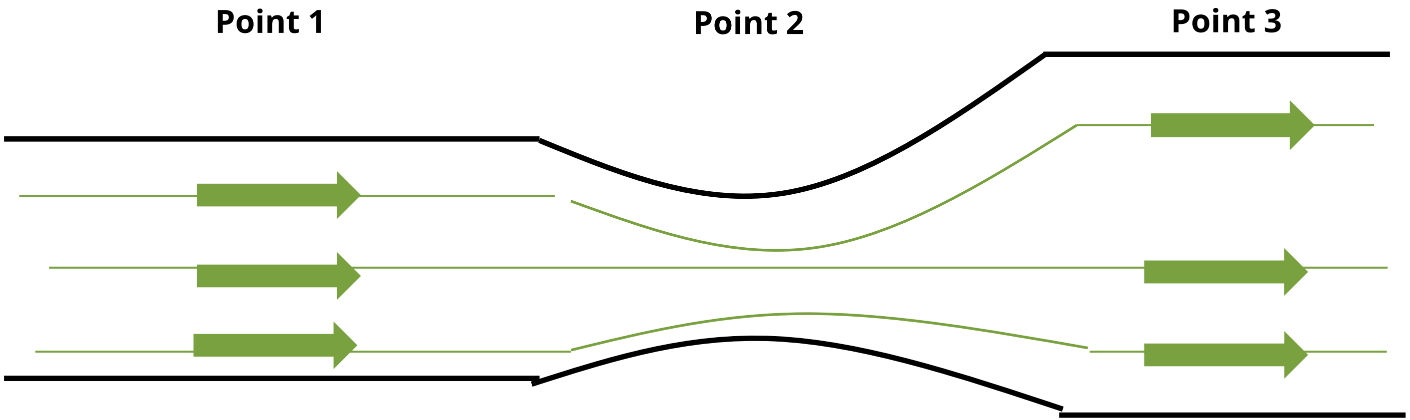
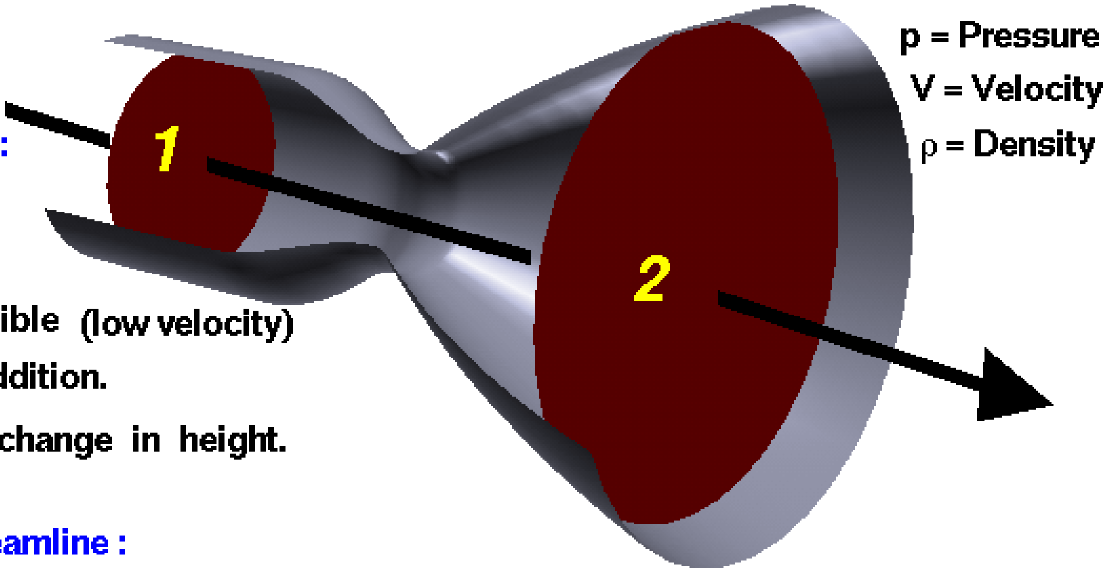


Image source: 2013 Emily Sappington, University of Houston

Bernoulli’s Equation



Restrictions :

Inviscid

Steady

Incompressible (low velocity)

No heat addition.

Negligible change in height.

Along a streamline :

static pressure + dynamic pressure = total pressure

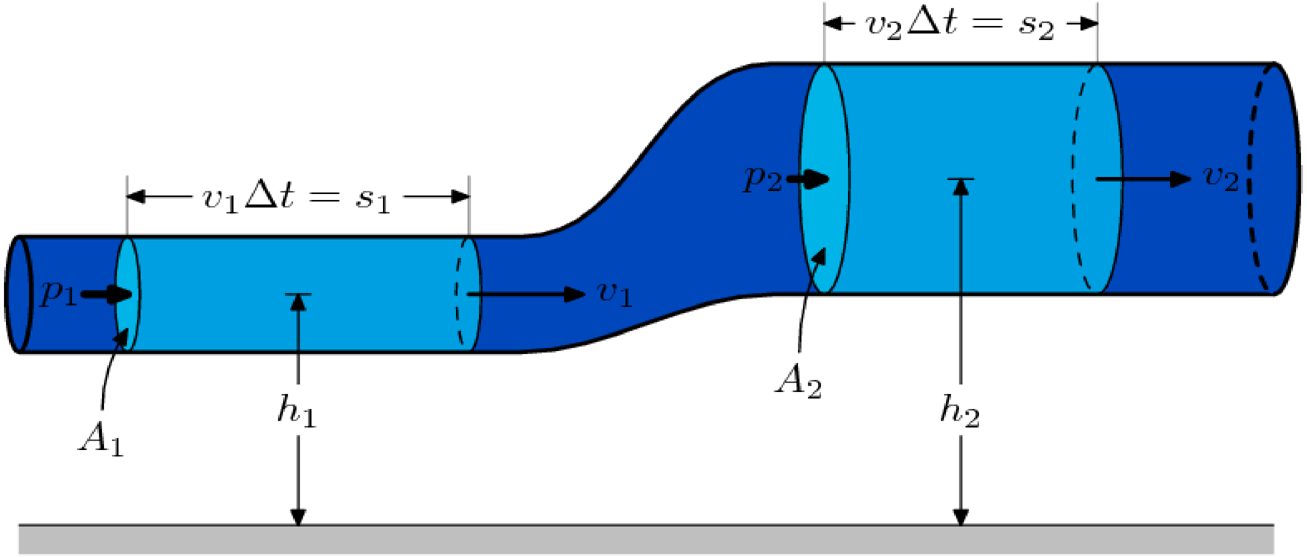
$$p_s + \frac{\rho V^2}{2} = p_t$$

$$\left(p_s + \frac{\rho V^2}{2}\right)_1 = \left(p_s + \frac{\rho V^2}{2}\right)_2$$

Source: NASA http://www.grc.nasa.gov/WWW/k-12/airplane/bern.html

Bernoulli's equation at two different points of varying height

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$



Source: http://commons.wikimedia.org/wiki/File:BernoullisLawDerivationDiagram.svg