Solid Mechanics Homework **Answers**

Please show all of your work, including which equations you are using, and circle your final answer. Be sure to include the units in your answers.

1. **The yield stress of steel is 250 MPa (250,000,000 Pa). A steel rod used for an implant in a femur needs to withstand 29 kN (29,000 N). What should the diameter of the rod be to not deform?**

   yield stress $(\sigma_y) = 250$ MPa
   $F = 29$ kN
   $d = ?$

   $$\sigma = \frac{F}{A}$$

   $$250,000,000 \, Pa = \frac{29,000 N}{A}$$

   $A = 1.16 x 10^{-4} \, m^2$
   $A = \pi r^2$
   $1.16 x 10^{-4} \, m^2 = \pi r^2$

   $3.69 x 10^{-5} \, m^2 = r^2$

   $\sqrt{3.69 x 10^{-5} \, m^2} = \sqrt{r^2}$

   $0.0061 m = r$

   $d = 2r$

   $d = 2 * 0.0061 m$

   $d = 0.0122 m \ or \ 12.2 mm$

2. **The yield stress of titanium is 830 MPa (830,000,000 Pa). A titanium rod used for an femur implant needs to withstand 29 kN (29,000 N). What should the diameter of the rod be to not deform?**

   yield stress $(\sigma_y) = 830$ MPa
   $F = 29$ kN
   $d = ?$

   $$\sigma = \frac{F}{A}$$

   $$830,000,000 \, Pa = \frac{29,000 N}{A}$$

   $A = 3.49 x 10^{-5} \, m^2$
   $A = \pi r^2$

$$3.49 x 10^{-5} m^2 = \pi r^2$$

$$1.11 x 10^{-5} m^2 = r^2$$

$$\sqrt{1.11 x 10^{-5} m^2} = \sqrt{r^2}$$

$$0.0033m = r$$

$$d = 2r$$

$$d = 2 * 0.0033m$$

$$d = 0.0067m \text{ or } 6.7mm$$

3. **Compare the rods from problems 1 and 2. Which rod can be smaller? What is the ratio between the steel rod diameter to the titanium rod diameter?**

The titanium rod can be smaller than the steel rod.

$$ratio = \frac{12.2mm}{6.7mm} = 1.82$$

The titanium rod can be almost half the size of the steel rod and still withstand the same force without deforming.

4. **The yield stress of steel is 250 MPa (250,000,000 Pa). A steel rod has a diameter of 13 mm (0.013 m). What is the maximum force that the rod can withstand?**

yield stress (σ_y) = 250 MPa
d = 13 mm
F = ?

$$\sigma = \frac{F}{A} \qquad A = \pi r^2 \qquad r = \frac{d}{2}$$

$$A = \pi \left(\frac{0.013m}{2} \right)^2$$

$$A = 1.327 x 10^{-4} m^2$$

$$250,000,000 Pa = \frac{F}{1.327 x 10^{-4} m^2}$$

$$F = 33183N \text{ or } 33.2kN$$

5. **The yield stress of titanium is 830 MPa (830,000,000 Pa). A titanium rod has a diameter of 13 mm (0.013 m). What is the maximum force that the rod can withstand?**

yield stress (σ_y) = 830 MPa
d = 13 mm
F = ?

$$\sigma = \frac{F}{A} \qquad A = \pi r^2 \qquad r = \frac{d}{2}$$

$$A = \pi \left(\frac{0.013m}{2} \right)^2$$

$$A = 1.327x10^{-4} \, m^2$$

$$830,000,000 Pa = \frac{F}{1.327x10^{-4} \, m^2}$$

$$F = 110167N \ or \ 110.2kN$$

6. **Compare the rods from problems 4 and 5. Which rod can withstand more force? What is the ratio between the titanium rod maximum force to the steel rod maximum force?**

The titanium rod can withstand more force before permanent deformation than the steel rod.

$$ratio = \frac{110.2kN}{33.2kN} = 3.32$$

The titanium rod can withstand more than three times the maximum force before permanent deformation compared to the steel rod.

7. **A steel rod that is 5.5 ft long stretched 0.04 in when a 2-kip (2,000 lb) tensile load is applied to it. Knowing that E = 29,000,000 psi, determine (a) the smallest diameter rod that should be used, (b) the corresponding stress caused by the load.**

$L = 5.5 \ ft \rightarrow 5.5 \ ft * 12 \ in = 66 \ in$

$\Delta L = 0.04 \ in$

$F = 2 \ kip$

$E = 29,000,000 \ psi$

$d = ?$

$\sigma = ?$

$$E = \frac{\sigma}{\varepsilon} \qquad\qquad \varepsilon = \frac{\Delta L}{L}$$

$$\varepsilon = \frac{0.04in}{66in}$$

$$\varepsilon = 6.06x10^{-4}$$

$$\sigma = E\varepsilon$$

$$\sigma = 29,000,000 \, psi * 6.06x10^{-4}$$

$$\sigma = 17574 \, psi$$

$$\sigma = \frac{F}{A} \qquad\qquad A = \pi r^2 \qquad d = 2r$$

$$17574\,psi = \frac{2000\,lb}{A}$$

$$A = 0.114\,in^2 = \pi r^2$$

$$0.036\,in^2 = r^2$$

$$\sqrt{0.036\,in^2} = \sqrt{r^2}$$

$$0.19\,in = r$$

$$d = 2r$$

$$d = 2*0.19\,in$$

$$d = 0.38\ in$$

8. **A 60-m-long steel wire is subjected to 6 kN (6,000 N) tensile force. Knowing that E = 200 GPa (200,000,000,000 Pa) and that the length of the rod increases by 48 mm (0.048 m), determine (a) the smallest diameter that may be selected for the wire, (b) the corresponding stress.**

$L = 60$ m
$F = 6$ kN
$E = 200$ GPa
$\Delta L = 48$ mm
$d = ?$
$\sigma = ?$

$$E = \frac{\sigma}{\varepsilon} \qquad\qquad \varepsilon = \frac{\Delta L}{L}$$

$$\varepsilon = \frac{0.048m}{60m}$$

$$\varepsilon = 8x10^{-4}$$
$$\sigma = E\varepsilon$$

$$\sigma = 200x10^9\,Pa*8x10^{-4}$$

$$\sigma = 160x10^6\ Pa\ or\ 160MPa$$

$$\sigma = \frac{F}{A} \qquad\qquad A = \pi r^2 \qquad\qquad d = 2r$$

$$160x10^6\,Pa = \frac{6000N}{A}$$

$$A = 3.75x10^{-5}\,m^2 = \pi r^2$$

$$1.19x10^{-5}\,m^2 = r^2$$

$$\sqrt{1.19x10^{-5}\,m^2} = \sqrt{r^2}$$

$$0.0035m = r$$

$$d = 2r$$
$$d = 2 * 0.0035m$$
$$d = 0.0069 \; m \; or \; 6.9mm$$

9. **A control rod made of yellow brass must not stretch more than 0.125 in when the tension in the wire is 800 lbs. Knowing that E = 15,000,000 psi and that the maximum allowable stress is 32 ksi (32,000 psi), determine (a) the smallest diameter that can be selected for the rod, (b) the corresponding maximum length of the rod.**

$\Delta L = 0.125$ in
$F = 800$ lb
$E = 15,000,000$ psi
$\sigma = 32$ ksi
$d = ?$
$L = ?$

$$\sigma = \frac{F}{A} \qquad A = \pi r^2 \qquad d = 2r$$

$$32,000 psi = \frac{800 lb}{A}$$

$$A = 0.025 in^2 = \pi r^2$$

$$0.0079 in^2 = r^2$$

$$\sqrt{0.0079 in^2} = \sqrt{r^2}$$

$$0.089 in = r$$

$$d = 2r$$

$$d = 2 * 0.089 in$$

$$d = 0.178 \; in$$

$$E = \frac{\sigma}{\varepsilon}$$

$$15,000,000 psi = \frac{32,000 psi}{\varepsilon}$$

$$\varepsilon = 0.002$$

$$\varepsilon = \frac{\Delta L}{L}$$

$$0.002 = \frac{0.125 in}{L}$$

$$L = 58.6 in$$

10. **A square aluminum bar should not stretch more than 1.4 mm when it is subjected to a tensile load. Knowing that E = 70 GPa (70,000,000,000 Pa) and that the allowable tensile strength is 120 MPa (120,000,000 Pa), determine (a) the maximum allowable length of the pipe, (b) the required dimensions of the cross section if the tensile load is 28 kN (28,000 N).**

$\Delta L = 48$ mm
$E = 70$ GPa
$\sigma = 120$ MPa
$F = 28$ kN
$dimensions = ?$
$L = ?$

$$E = \frac{\sigma}{\varepsilon}$$

$$70x10^9 Pa = \frac{120x10^6 Pa}{\varepsilon}$$

$$\varepsilon = 0.0017$$

$$\varepsilon = \frac{\Delta L}{L}$$

$$0.0017 = \frac{0.0014m}{L}$$

$$L = 0.816m \ or \ 816.7mm$$

$$\sigma = \frac{F}{A}$$

$$120x10^6 Pa = \frac{28,000N}{A}$$

$$A = 2.33x10^{-4} m^2$$

$$A = b^2$$

$$2.33x10^{-4} m^2 = b^2$$

$$\sqrt{2.33x10^{-4} m^2} = \sqrt{b^2}$$

$$b = 0.0153m \ or \ 15.3mm$$

$$dimensions = 15.3mm \ x \ 15.3mm$$

11. **Use the engineering stress-strain diagram for tensile tests of metals A and B to answer the following questions. Each test sample is 10 mm in diameter with a gage length of 50mm.**
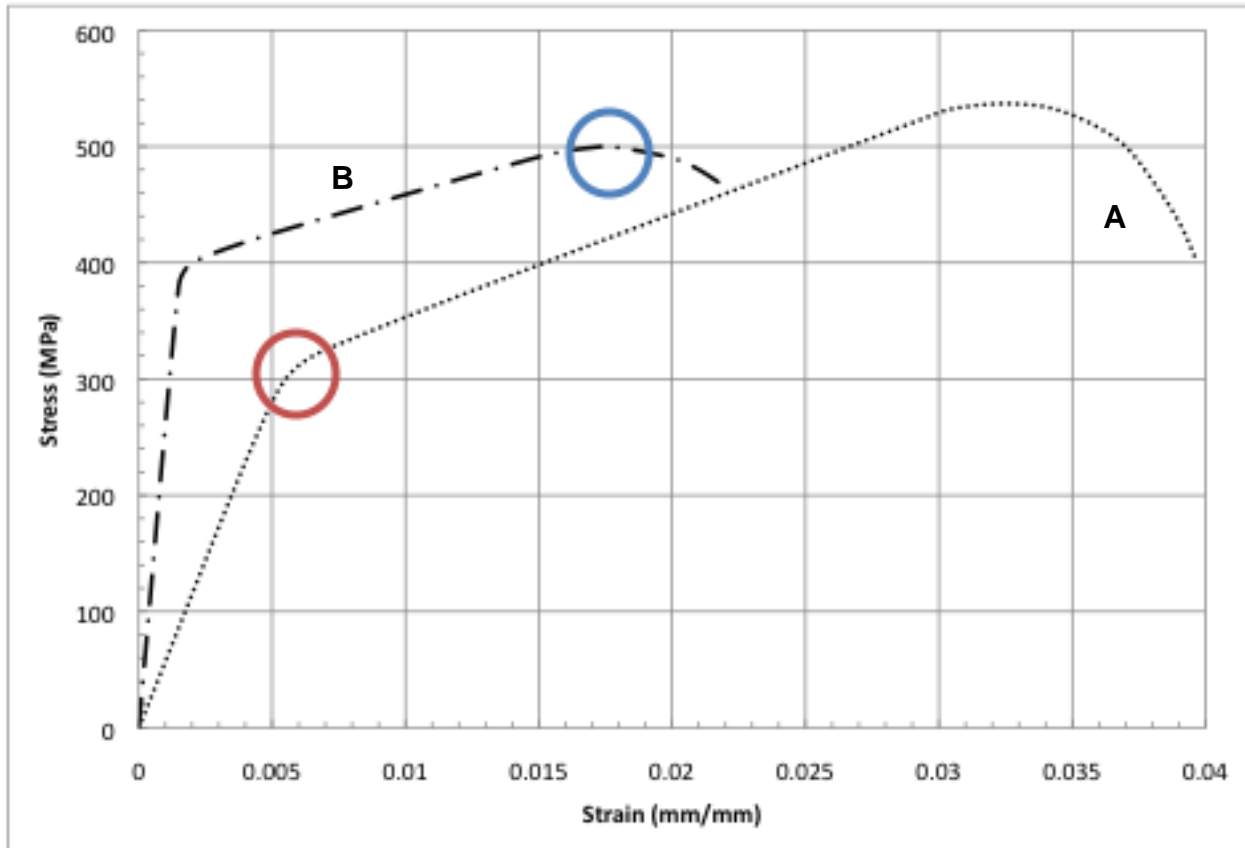
a. **Which material has the lowest yield stress? What is the value? Label the yield point for this material on the graph.**

Metal A has the lowest yield stress. It is approximately 310 MPa (labeled in red, above).

b. **Which material has the lowest ultimate tensile strength? What is the value? Label the ultimate tensile strength for this material on the graph.**

Metal B has the lowest ultimate tensile strength. It is approximately 500 MPa (labeled in blue, above).

c. **Which material has a larger modulus of elasticity?**

Metal B has a larger modulus of elasticity (larger slope of the initial linear region).